Storing Heat in the Ground

OK, Here's a simplistic calculation the rate that we have to supply heat to a ground store to compensate for its losses. Call this rate *Q* (watts).

Let's use David O's figure of 2000 m^3 storage at a temperature difference of 5 K .
Let's assume a hemisphere of earth under the house for simplicity.

Then the volume is given by $V=\frac{2}{3}\pi r^3$ so the radius *r* is 14.42 m.

And the area of the boundary *A* is $2\pi r^2$ m^2.

All the earth in the hemisphere is at least ΔT degrees above ambient soil temperature and we assume that the boundary is isothermal at ΔT so we're not wasting energy.

We ignore all heat loss at the earth surface (perfect insulation everywhere!) and we assume the heat propagation is isotropic.

The heat loss per unit area through the boundary is *Q/A* (W/m^2)

Suppose the thermal conductivity of the soil is *k* (W/m.K) and that it is constant everywhere and at all temperatures of interest.

By Fourier's Law $\frac{Q}{A}=-k\nabla T$ and since we have spherical symmetry, ∇T is just $\frac{dT}{dr}$ (K/m)

Substituting, $\frac{Q}{A}=\frac{Q}{2\pi r^2}=-k\frac{dT}{dr}$ and integrating and using the boundary condition at infinity we find $\Delta T=\frac{Q}{2\pi k R}$ which can be rearranged as $Q=2\pi k R\Delta T$

We now just need to make numerical substitutions. The conductivity of soils has quite a wide range depending on its type, compaction, moisture content etc. Let's take 0.3 W/m.K as a lower bound and 2 W/m.K as an upper bound.

With those values we get a range for the heat loss power:

$$136\,W\leq Q\leq 906\,W$$

I think those are low estimates because we were optimistic about not losing any heat upwards and about the spherical symmetry but I also have another concern.

David O suggested a temperature differential of 5 K, which I used in the calculations above. But I think that to be directly useful to warm a house, the store needs to be at a temperature of at least 5 K warmer than **the house** at say 20 $^\circ$C whilst the heat is lost at the ambient soil temperature of around 10 $^\circ$C, so I believe we need to consider the difference ΔT needs to be at least 15 K.

If we repeat the calculations, we get a range of $408\,W\leq Q\leq 2718\,W$

That's the continuous power that needs to be supplied.